Teacher notes
Topic A

Two nice problems on projectile motion

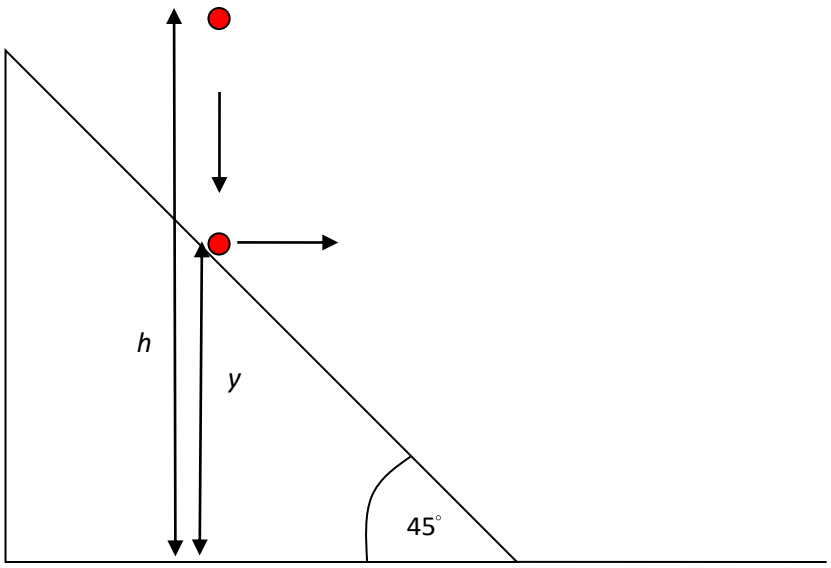
1. At what angle should a projectile be thrown so that its maximum height is the same as the horizontal distance travelled?

We know that $v_y = u\sin\theta - gt$ so the projectile reaches the maximum height when $0 = u\sin\theta - gt \Rightarrow t = \frac{u\sin\theta}{g}$ and falls to the ground in time $t = \frac{2u\sin\theta}{g}$. Hence the horizontal distance travelled is $x_{\max} = u\cos\theta\frac{2u\sin\theta}{g} = \frac{2u^2\cos\theta\sin\theta}{g}$ and

$$y_{\max} = u\sin\theta\frac{u\sin\theta}{g} - \frac{1}{2}g\left(\frac{u\sin\theta}{g}\right)^2 = \frac{u^2\sin^2\theta}{g} - \frac{u^2\sin^2\theta}{2g} = \frac{u^2\sin^2\theta}{2g}.$$

$$\frac{2u^2\cos\theta\sin\theta}{g} = \frac{u^2\sin^2\theta}{2g}$$
$$4\cos\theta = \sin\theta$$
$$\tan\theta = 4$$
$$\theta \approx 76^\circ$$

2. A ball is dropped from rest at a height *h* above level ground. The ball bounces off a surface inclined at 45° to the horizontal at a height *y* from level ground. The speed is unchanged at impact. The height *h* is sufficiently large that the ball lands on horizontal ground. What is *y* such that the ball travels the maximum horizontal distance?

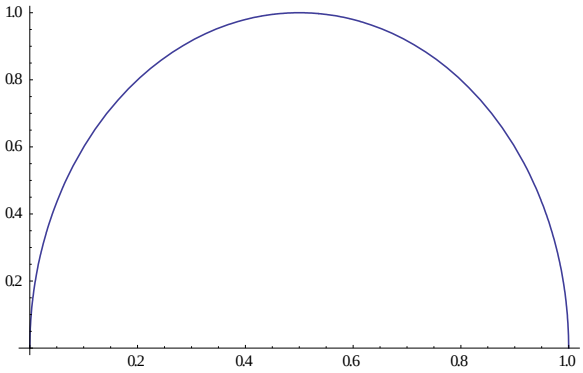


The ball will hit the incline with speed given by $v^2 = 2g(h-y)$.

Because there is no change in speed the ball will bounce horizontally. Hence it will hit horizontal ground in a time given by $y = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2y}{g}}$.

The horizontal distance travelled is then $x = v\sqrt{\frac{2y}{g}} = \sqrt{2g(h-y)}\sqrt{\frac{2y}{g}} = 2\sqrt{(h-y)y}$.

We can write this as $x = 2h\sqrt{(1-\frac{y}{h})\frac{y}{h}}$. Plotting $2\sqrt{(1-\frac{y}{h})\frac{y}{h}}$ as a function of y/h we get:



This is a maximum when $\frac{y}{h} = \frac{1}{2}$, i.e., $y = \frac{h}{2}$, giving $x_{\max} = 2\sqrt{(h-\frac{h}{2})\frac{h}{2}} = h$.